Medical Image Segmentation Based on Variational Bayes

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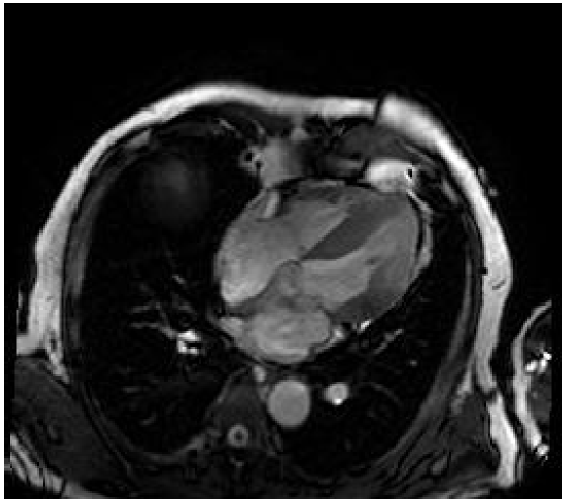
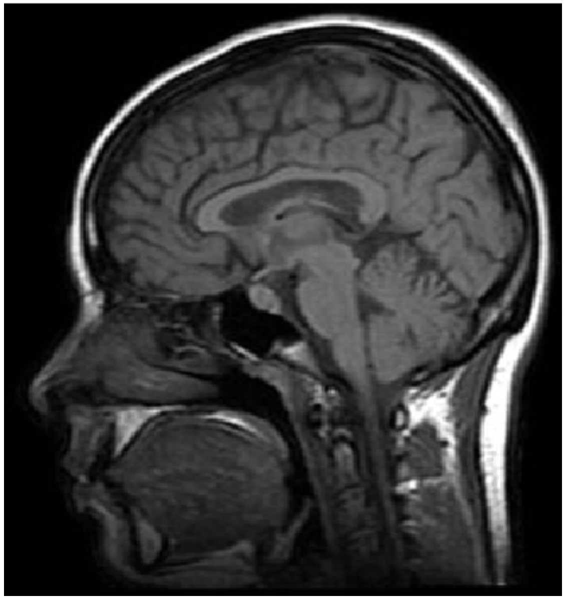
Outline

- Motivation
- Brief introduction of Variational Bayes
- Variational Inference for Mixture model
  - Mixture of Gaussian model
  - Finite Student's t-mixture
  - Infinite Student's t-mixture
  - Experiment
- Laplacian Regularized Gaussian mixture model
  - Laplacian Regularization
  - Experiment
- Summary

Motivation

- Application
  - Medical Image analysis
  - 3D reconstruction
  - Data compression
  - Image understanding
- Approach
  - Region segmentation
  - Edge-Detection
  - Markov random Field
  - Clustering-Based (mixture model)





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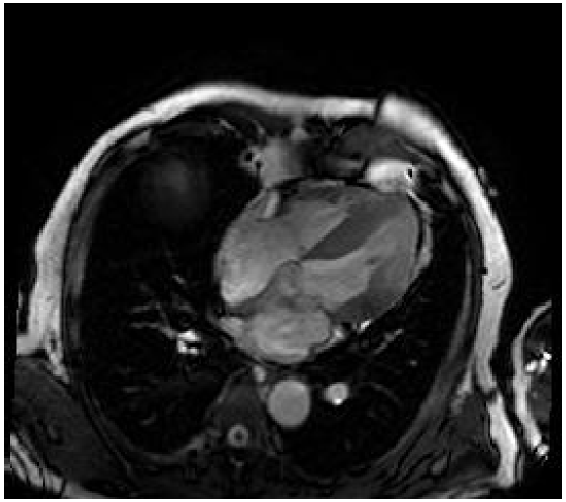

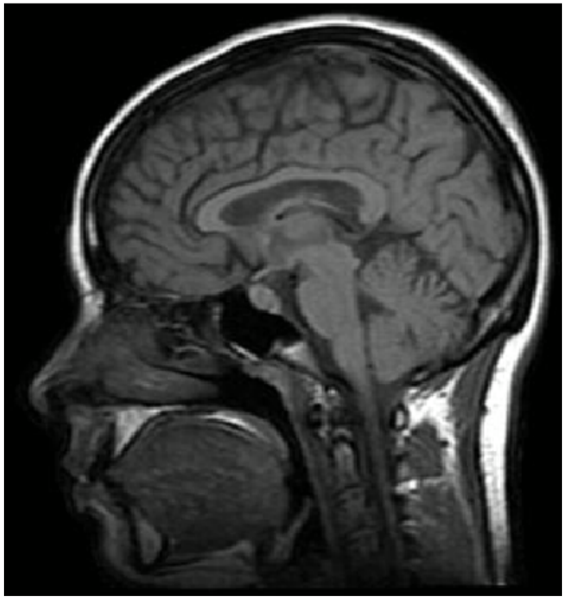

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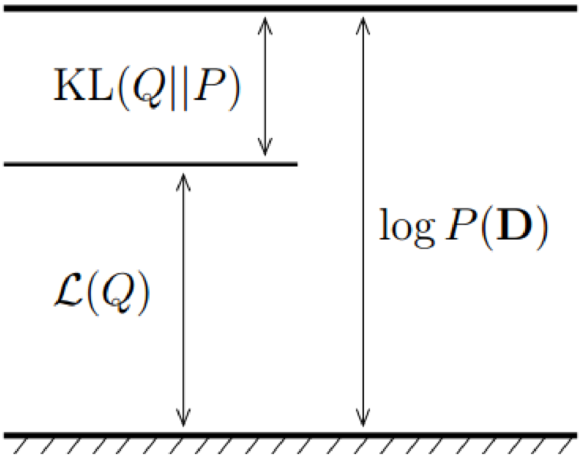
Variational Bayes



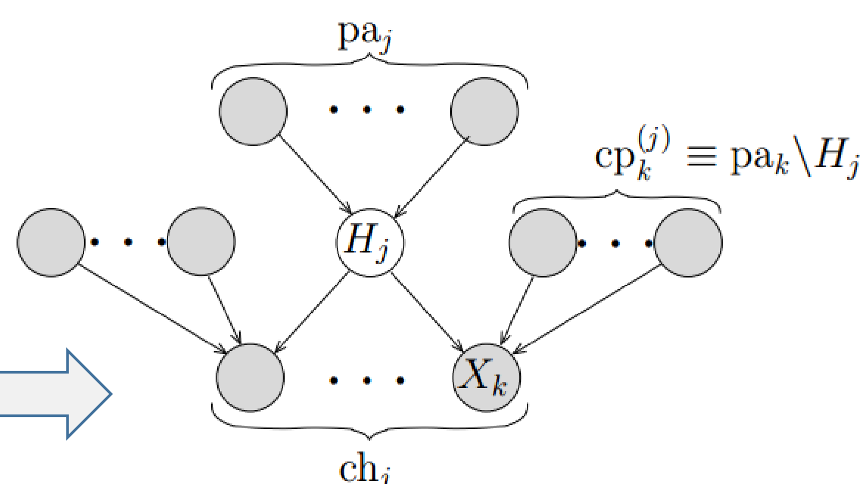
Key Notes:

- Distributional Approximation
- By minimizing the KL divergence.
- Mean Field assumption
- Bayesian frameworks

Mean-field Assumption

Variational Methods

$$Q(Z_i) \propto \frac{1}{C} \exp \left\langle \ln P(Z_i, Z_{-i}, D) \right\rangle_{Q(Z_{-i}) or Q(mb(Z_i))}$$



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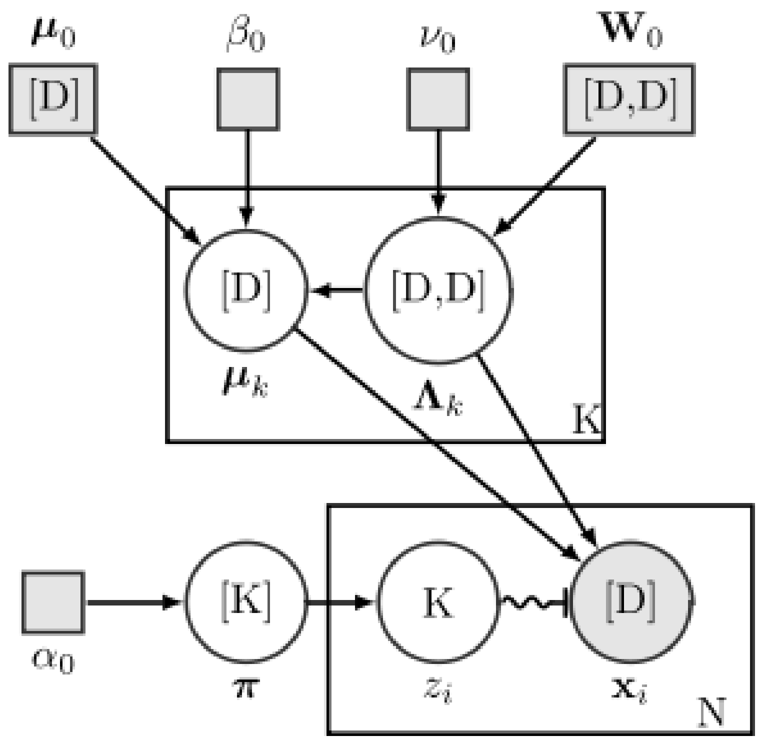
mixture of Gaussian

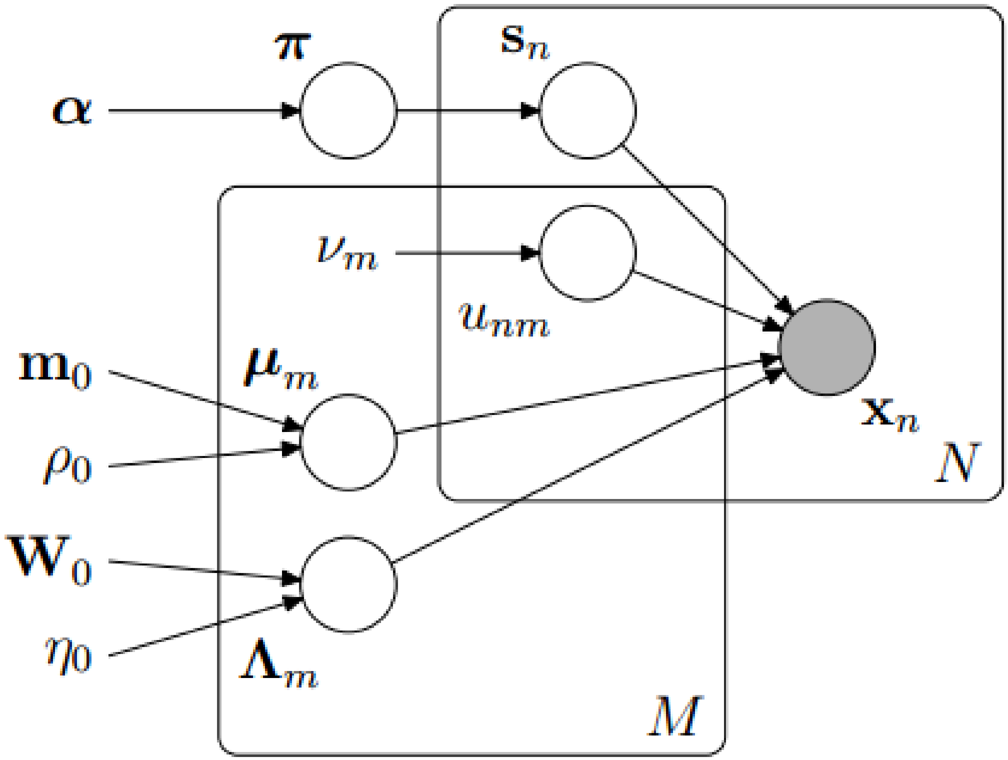
$$p(X \mid Z, \mu, \Lambda) = \prod_{n=1}^{N} \prod_{k=1}^{K} N(x_n \mid \mu_k, \Lambda_k^{-1})^{z_{nk}}$$

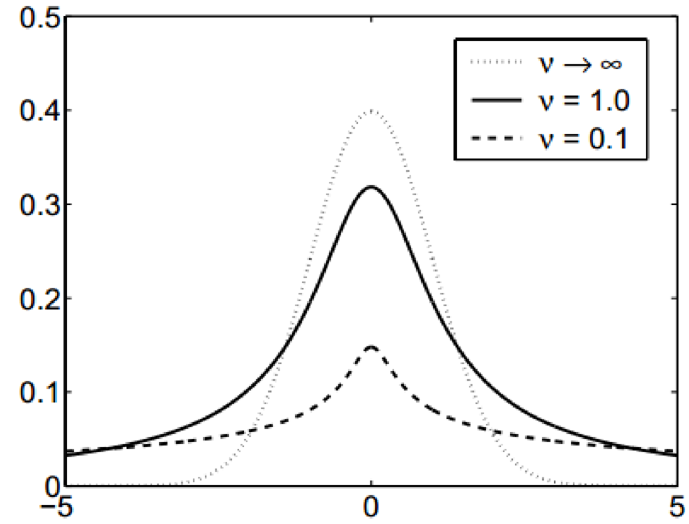


$$p(X, Z, \pi, \mu, \Lambda) = p(X \mid Z, \mu, \Lambda)\, p(Z \mid \pi)\, p(\pi)\, p(\mu \mid \Lambda)\, p(\Lambda)$$

Student's t-mixture model

$$p(x_n \mid s, \{\mu, \Lambda, v\}) = \sum_{m=1}^{M} St(x_n \mid \mu_m, \Lambda_m, v_m)^{s_m}$$





Infinite Student's t-mixture

$$DP(\alpha, G_0)$$

Dirichlet Process

$$G = \sum_{j=1}^{\infty} \pi_j(V)\delta_{\Theta_j} \quad \pi_j(V) = V_j \prod_{i=1}^{j-1}(1-V_i)$$

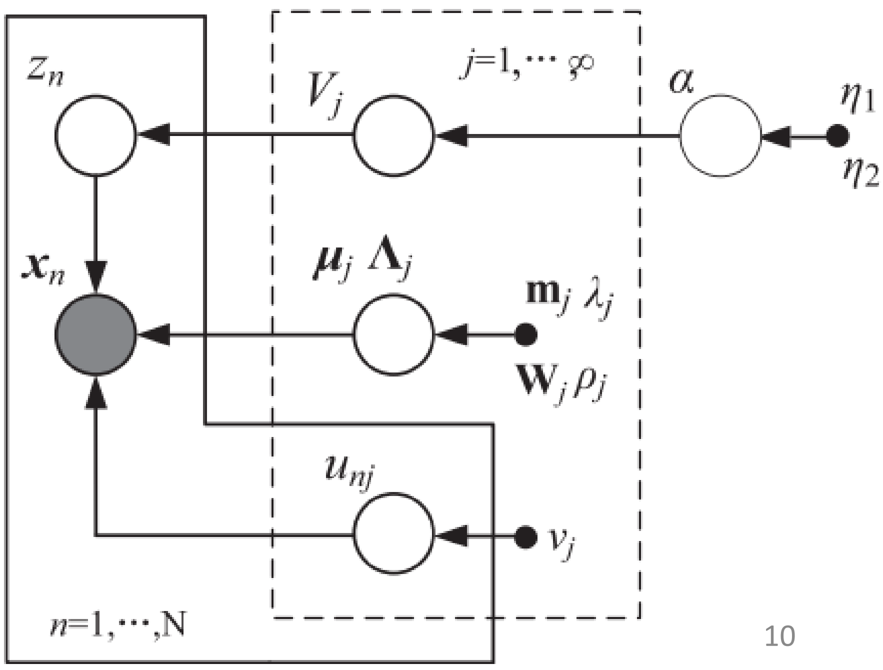
Stick-Breaking prior

$$V_j \sim Beta(1, \alpha)$$

$$p(\alpha) = Gam(\alpha \mid \eta_1, \eta_2)$$

Dirichlet Process Mixture

$$p(X) = \prod_{n=1}^{N} \sum_{j=1}^{\infty} \pi_j(V) \cdot St(x_n \mid \mu_j, \Lambda_j, v_j)$$



Example: Variational Inference for GMM

$$q^*(\pi) \sim Dir(\alpha) \qquad \alpha = \alpha_0 + N_k \qquad N_k = \sum_{n=1}^{N} r_{nk}$$

Soft-count or ESS

$$q^*(\mu_k, \Lambda_k) = N(\mu_k \mid m_k, (\beta_k \Lambda_k)^{-1}) W(\Lambda_k \mid w_k, v_k)$$

$$\beta_k = \beta_0 + N_k, m_k = \frac{1}{\beta_k}(\beta_0 m_0 + N_k \overline{x}_k), v_k = v_0 + N_k, \overline{x}_k = \frac{1}{N_k}\sum_{n=1}^{N} r_{nk} x_n$$

$$w_k^{-1} = w_0^{-1} + N_k S_k + \frac{\beta_0 N_k}{\beta_0 + N_k}(\overline{x}_k - m_0)(\overline{x}_k - m_0)^T, S_k = \frac{1}{N_k}\sum_{n=1}^{N} r_{nk}(\overline{x}_k - x_n)(\overline{x}_k - x_n)^T$$

VBM-Step

VBE-Step

Latent variable

$$q^*(Z) = \prod_{n=1}^{N}\prod_{k=1}^{K} r_{nk}^{z_{nk}} \qquad r_{nk} \propto \tilde{\pi}_k \tilde{\Lambda}_k^{1/2} \exp\left\{-\frac{D}{2\beta_k} - \frac{v_k}{2}(x_n - m_k)^T W_k (x_n - m_k)\right\}$$

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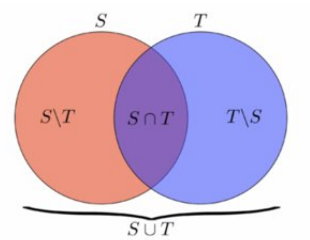
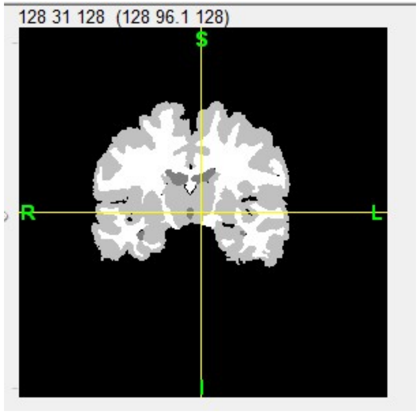
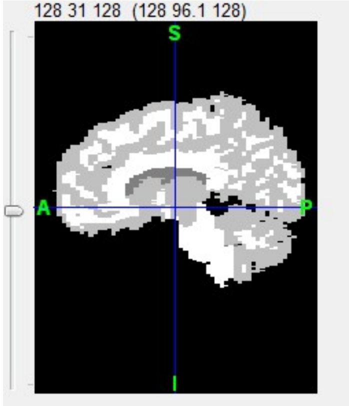
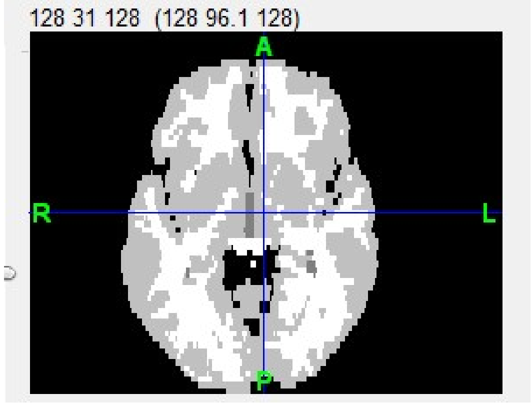
Experiment

- Data: Internet Brain Segmentation Repository(IBSR) [1], including 20 low resolution T1-weighted brain MRI image.
- Task: segment MRI into Gray Matter (GM), White Matter (WM) and Cerebrospinal Fluid (CSF)
- Measure: Jaccard similarity coefficient (JSC)
- MATLAB toolbox (preprocessing)：SPM8





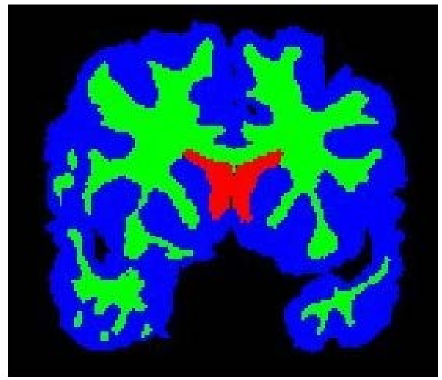




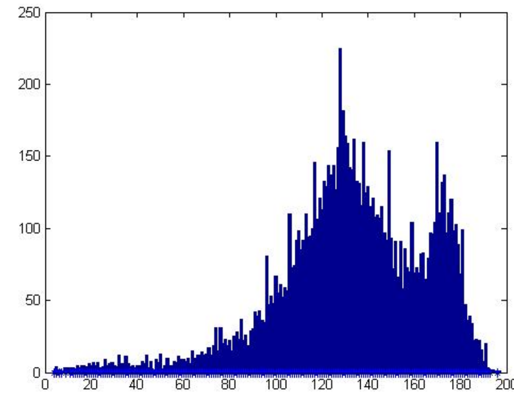
1. Center for Morphometric Analysis at Massachusetts General Hospi-tal, "The Internet Brain Segmentation Repository (IBSR),"http://www.cma.mgh.harvard. edu/ibsr/index.html, Jan. 2009

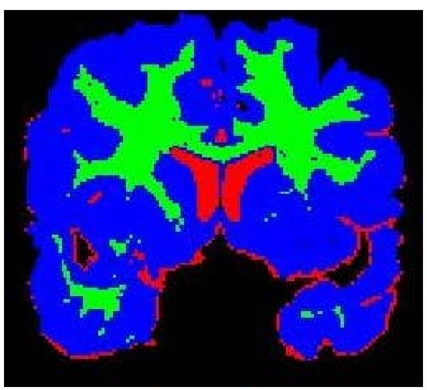
Segmentation Result

Source Data

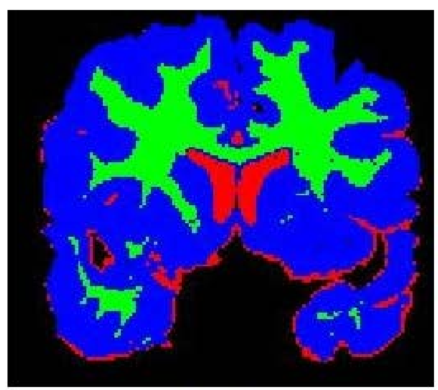


(a) groundTruth

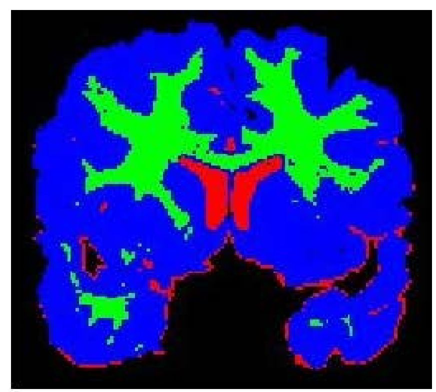




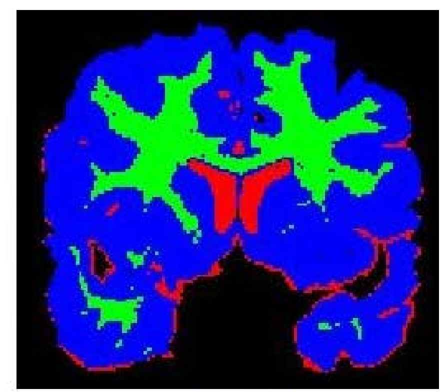
(b) EM-GMM



(c) VB-GMM

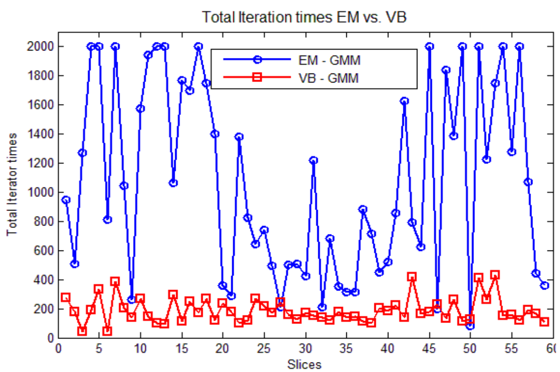


(d) VB-SMM

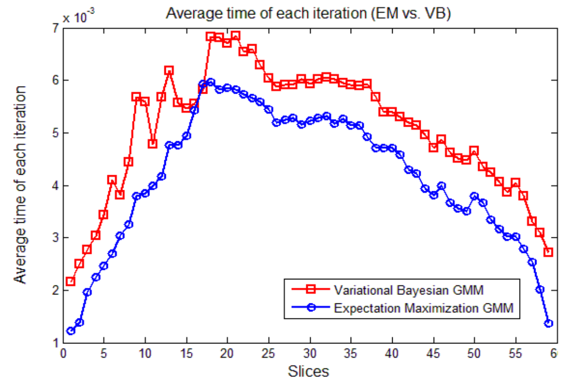


(e) VB-iSMM

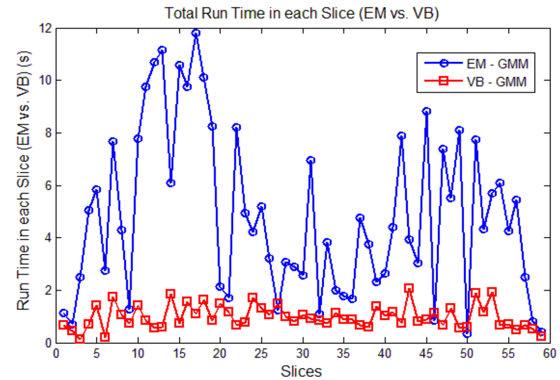
Time cost: EM (blue) vs. VB(red)



(a) Total Iteration times



(b) Average time of each iteration



(c) Total Run time

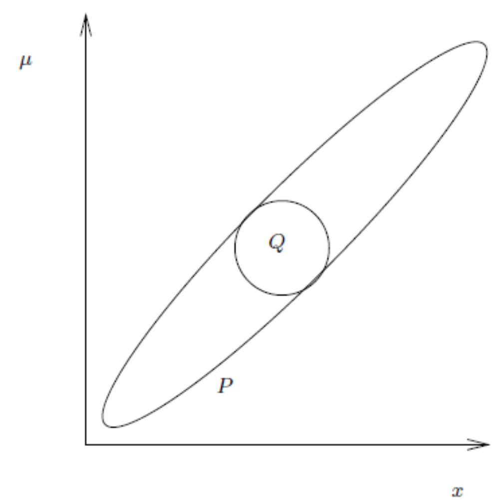
(a) Total iteration times (b) Time at each iteration (c) Total time

Algorithm complexity:

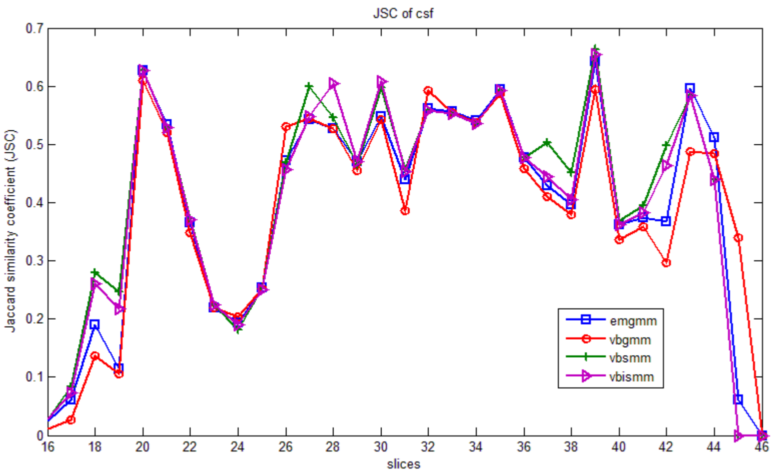
$$O(tKNd^3)$$

Iteration Times:

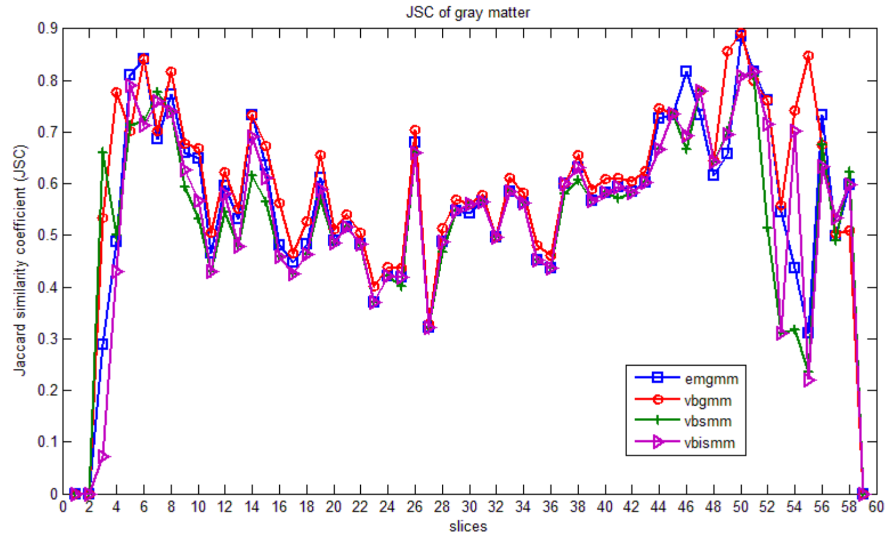
$$t_{VB} = (1/10 \sim 1/2)t_{EM}$$



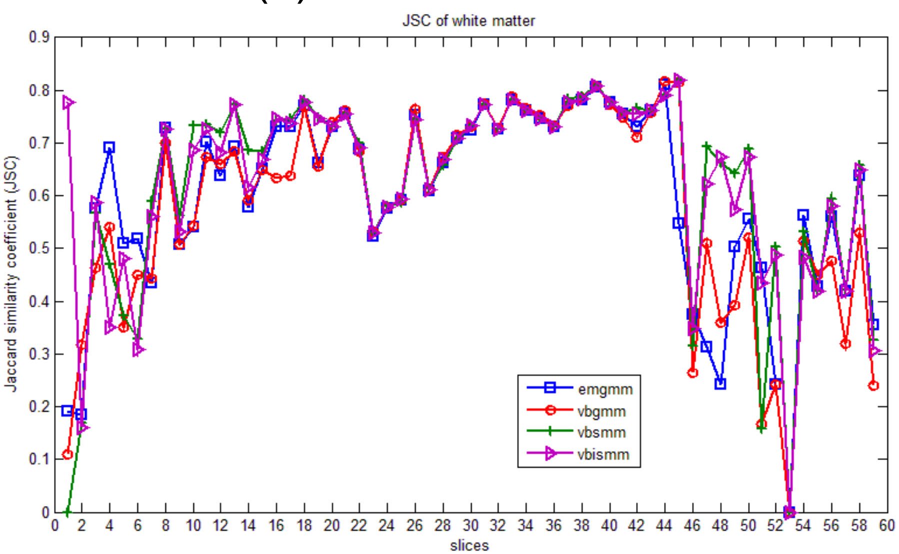
Segmentation Accuracy



(a) JSC of CSF



(b) JSC of GM



(c) JSC of WM

accuracy is not reduced.

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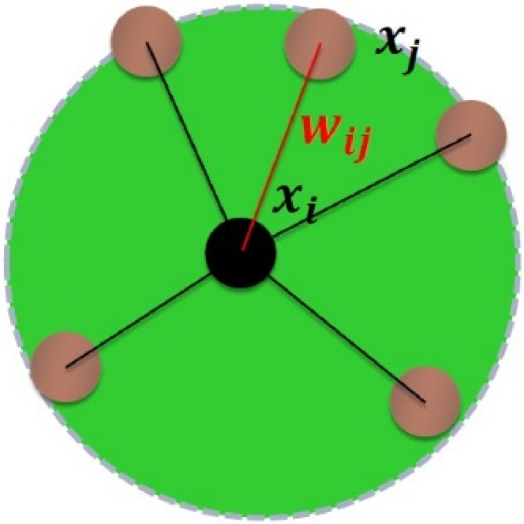
Laplacian regularization

- Manifold assumption.
- Construction
  - p-nearest neighbors graph
  - Assign weight matrix S
  - Laplacian graph: L=D-S

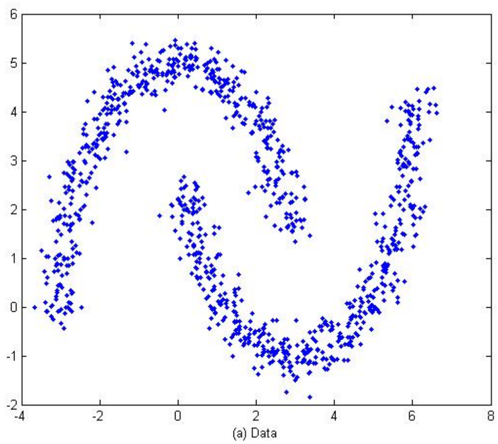


$$\min \quad R_k = \frac{1}{2}\sum_{i,j=1}^{m}(P(k \mid x_i) - P(k \mid x_j))^2 S_{ij}$$

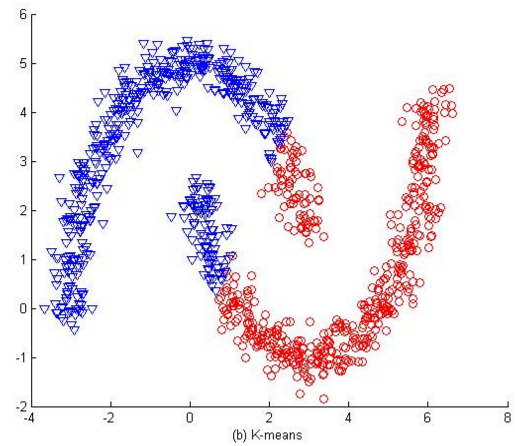
$$\max \quad L(\Theta) = \log P(X \mid \Theta) - \boxed{\lambda \sum_{k=1}^{K} R_k}$$

Regularization

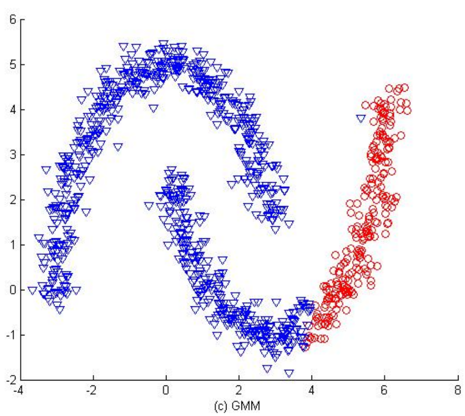
A Toy Example: Two Moons Pattern



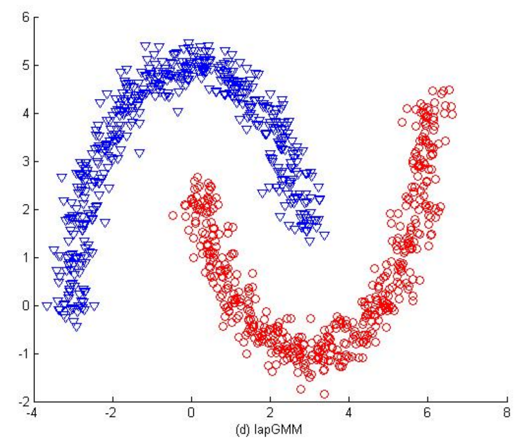
(a) Original data



(b) K-means



(c) VB-GMM



(d) VB-lapGMM

Segmentation Result

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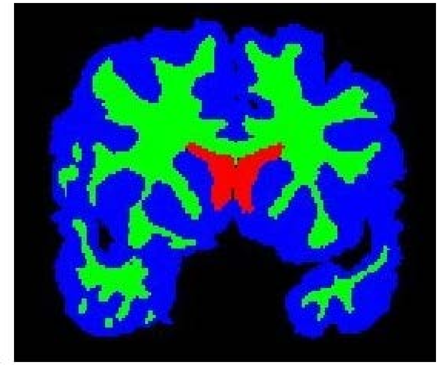

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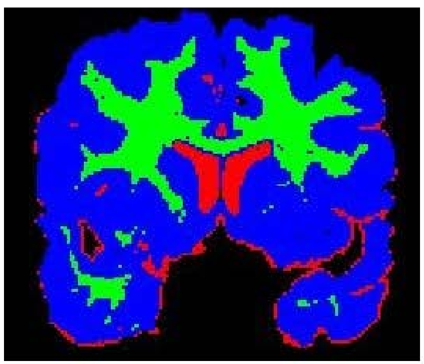

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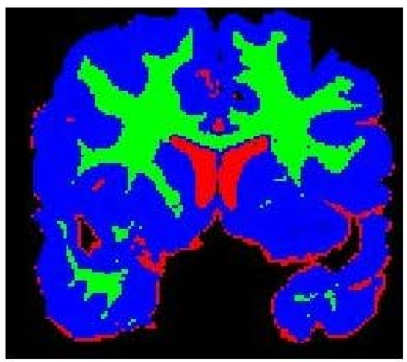

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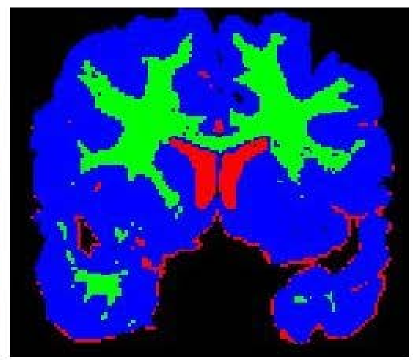
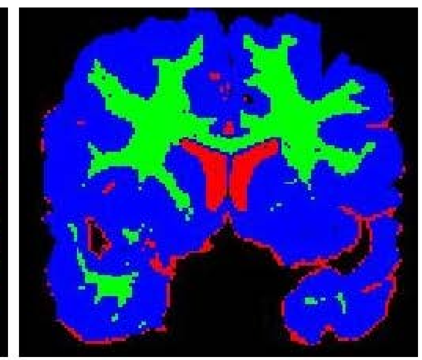
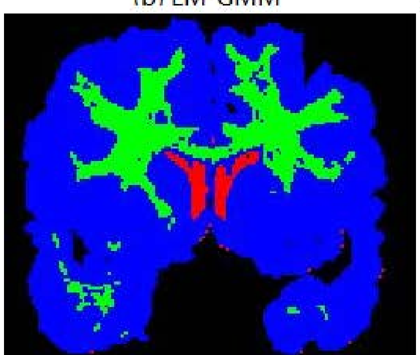


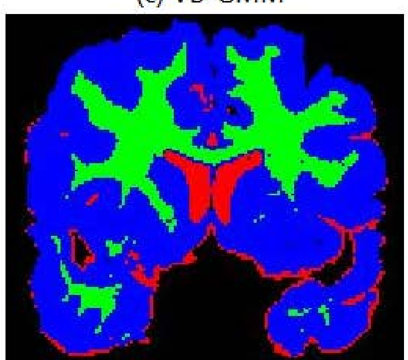
(d) VB-SMM



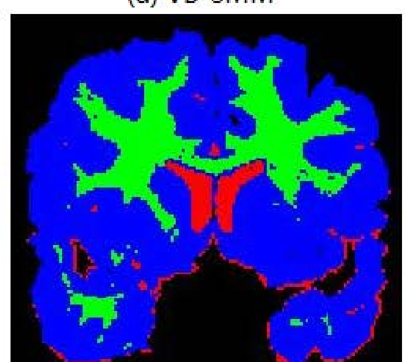
(e) VB-iSMM



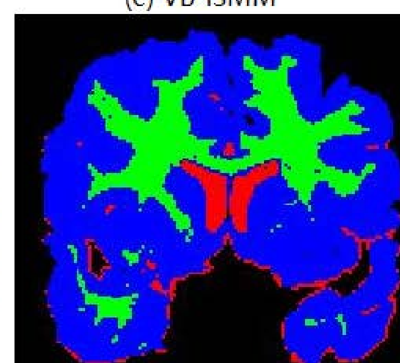
(f) EM-lapGMM



(g) VB-lapGMM

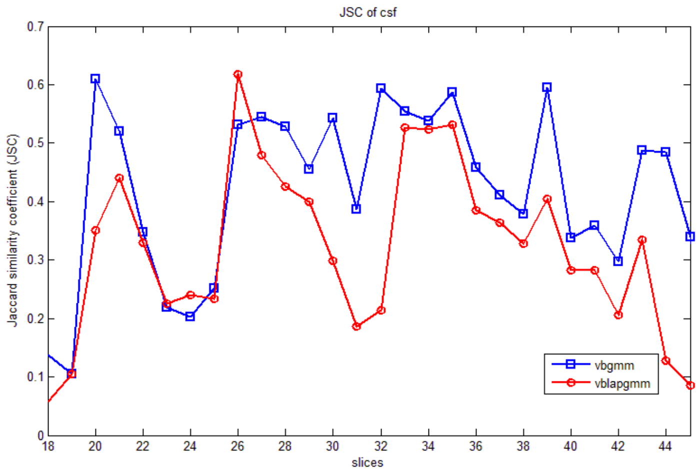


(h)VB-lapSMM

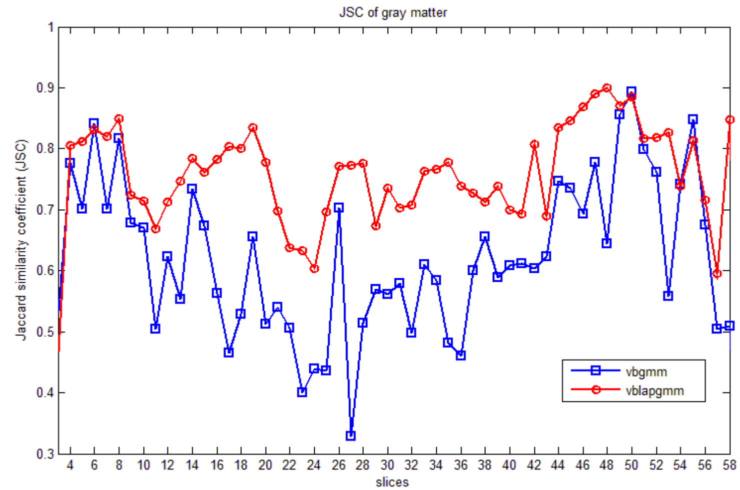


(i) VB-lapiSMM

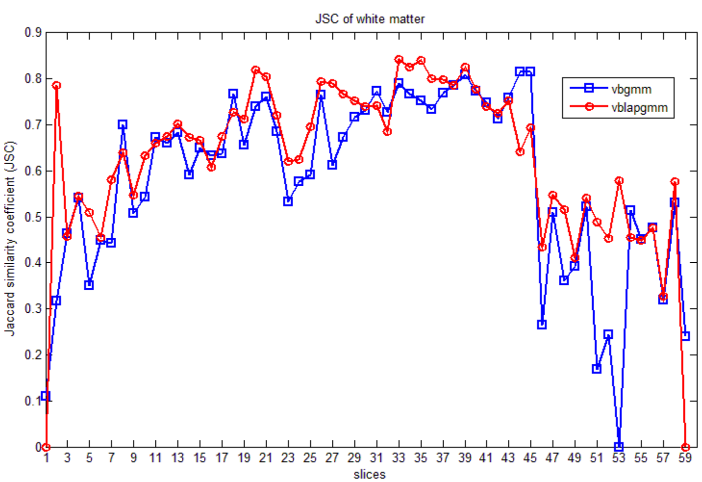
Accuracy: vbgmm vs. vblapgmm



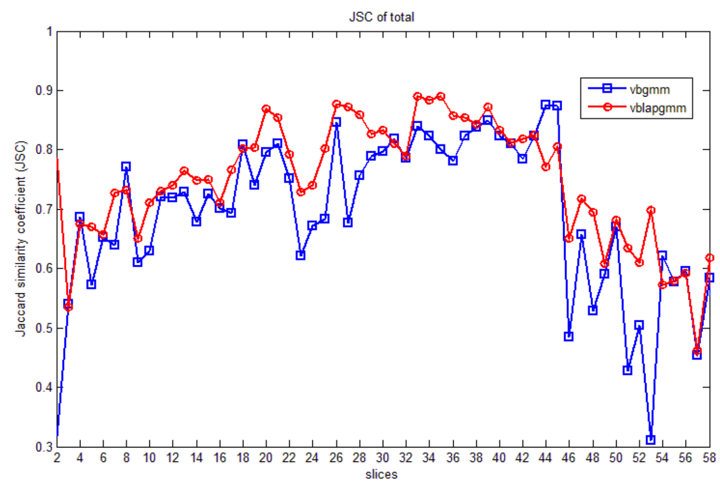
(a) Cerebrospinal Fluid



(b) Gray matter



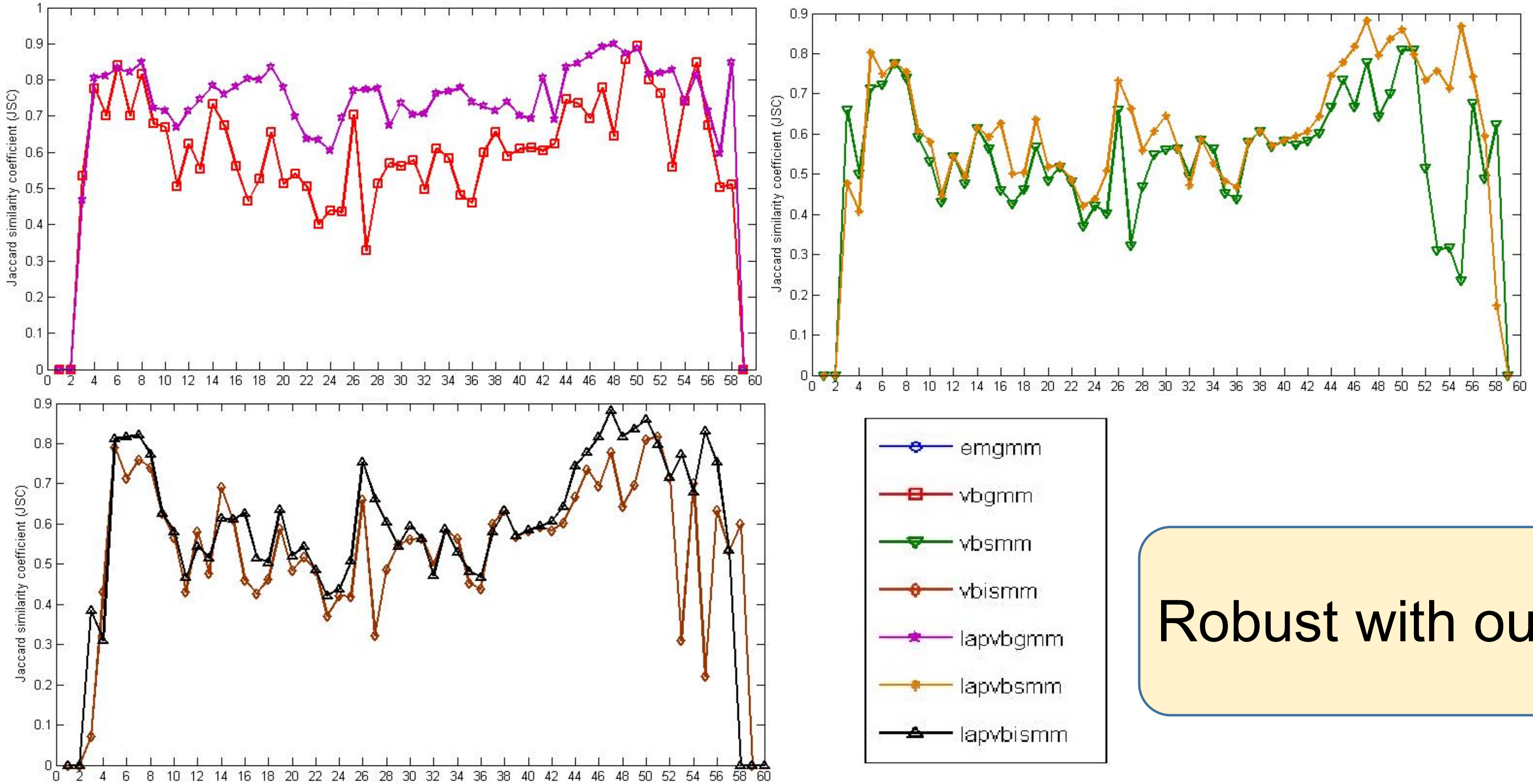
(c) White matter



(d) Total

Improve Accuracy

Result: SMM/iSMM vs. GMM



Robust with outlier

(a) top-left: VB-GMM vs. VB-lapGMM
(b) top-right: VB-SMM vs. VB-lapSMM
(c) lower-left: VB-iSMM vs. VB-lapiSMM

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Variational Bayes vs. Expectation Maximization

Reduce the iteration times, 1/10~1/2

finite/Infinte Students' t-mixture model

Reduce noise ，more robust

Variational laplacian regularized mixture model

Improve accuracy, enhance stability

Thank you